Tailoring the thermal Casimir forces in graphene systems: effect of electric current

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We investigate the dependence of the thermal Casimir force between two graphene sheets on the drift velocity of the electrons in one graphene sheet. We show that the drift motion produces a measurable change of the thermal Casimir force due to the Doppler effect. The thermal Casimir force as well as the Casimir friction are strongly enhanced in the case of resonant photon tunneling when the energy of the emitted photon coincides with the energy of electron-hole pair excitations. In the case of resonant photon tunneling, even for temperatures above room temperature the Casimir friction is dominated by quantum friction due to quantum fluctuations. Quantum friction can be detected in frictional drag experiment between graphene sheets for high electric field.

In the late 1940s Hendrik Casimir predicted[1] that two macroscopic non-magnetic bodies with no net electric charge (or charge moments) can experience an attractive force much stronger than gravity. The existence of this force is one of the few direct macroscopic manifestations of quantum mechanics; others are superfluidity, superconductivity, and the black body radiation spectrum.

Casimir based his prediction on a simplified model involving two parallel perfectly conducting plates separated by vacuum. A theory of the van der Waals and Casimir forces between parallel material plates in thermal equilibrium and separated by a vacuum gap was developed by Lifshitz (1955)[2]. Lifshitz's theory describes dispersion forces between dissipative media as a physical phenomenon caused by the fluctuating electromagnetic field that is always present in both the interior and the exterior of any medium. Outside the medium this field exists partly in the form of the radiative propagating waves, and partly in the form of nonradiative evanescent waves whose amplitudes decay exponentially with the distance away from the medium. To calculate the fluctuating electromagnetic field Lifshitz used Rytov's theory[3–5]. Rytov's theory is based on the introduction into the Maxwell equation of a "random" field (just as, for example, one introduces a "random" force in the theory of Brownian motion). Both quantum and thermal fluctuations give contributions to the total Casimir force. A general theory of Casimir and van der Waals forces was developed in Ref.[6] using quantum field theory. This theory confirmed the results of Lifshitz's theory. Quantum fluctuations dominate at small separation ($d < \lambda_T = c\hbar/k_BT$) and thermal fluctuations dominate at large separation ($d > \lambda_T$). Casimir forces due to quantum fluctuations have been studied experimentally for a long time[7,8]. However the Casimir forces due to thermal fluctuations were measured only recently, and the results confirmed the prediction of the Lifshitz theory[9]. At present the interest in Casimir forces is increasing because they dominate the interaction between nanostructures and are responsible for the adhesion between moving parts in small devices such as micro- and nano-electromechanical systems[10,11]. Due to this practical interest and the fast progress in force detection techniques, experimental[9,12–15] and theoretical[16,17] investigations of Casimir forces have experienced an extraordinary "renaissance" in the past few years. At present a great deal of attention is devoted to the study of the Casimir forces in graphene systems[18–26].

Graphene, isolated monolayer of carbon, which was obtained very recently, consists of carbon atoms densely packed into a two-dimensional honeycomb crystal lattice. The unique electronic and mechanical properties of graphene are actively studied both theoretically, and experimentally because of their importance for fundamental physics, and also possible technological applications[27–29]. In particular, the valence band and conduction band in graphene touch each other at one point named the Dirac point. Near this point the energy spectrum for electrons and holes has a linear dispersion. Due to this linear (or "conical") dispersion relation electrons and holes near these point behave like relativistic particles described by the Dirac equation for massless fermions. Because of the unusual electronic properties of graphene, the Casimir forces in graphene also have unusual properties. Contribution to the Casimir force due to thermal fluctuations for normal materials dominates for $d > \gamma_T = \hbar c/k_BT$ but for two graphene sheets thermal contribution dominates for much shorter distances[26] $d > \xi_T = \hbar v_F/k_BT$, where $v_F \sim 10^6$ m/s is the Fermi velocity in graphene. At room temperature the parameters γ_T and ξ_T are 25 nm and 7.6 μm, respectively. This property makes it possible to measure the thermal Casimir force using an atomic force microscope or other force measuring techniques. Tailoring the thermal Casimir force using Fermi level tuning by gate voltage was discussed in

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Ref.[24]. Alternative method of tailoring the thermal Casimir force consists in driving an electric current in a graphene sheet. Due to the high mobility of carriers in graphene, in a high electric field electrons (or holes) can move with very high velocities (up to 10^6 m/s). The drift motion of electrons will result in a modification of dielectric properties (and the Casimir force) of graphene due to the Doppler effect[30]. If in one of two parallel graphene sheets an electric current is induced, then the electromagnetic waves, radiated by the graphene sheet without an electric current, will experience a frequency Doppler shift in the reference frame moving with the drift velocity v of electrons in the other graphene sheet: $\omega' = \omega + q_x v$, where q_x is the parallel to the surface component of momentum transfer. The same is true for the waves emitted by the other graphene sheet. Due to the frequency dependence of the reflection amplitudes the electromagnetic waves will reflect differently in comparison to the case when there is no drift motion of electrons, and this will give rise to the change of the Casimir force.

In this Letter, we investigate the dependence the thermal Casimir force between graphene sheets on the drift velocity v of charge carriers in one of the graphene sheet. Let us consider two graphene sheets at separation $d \ll \lambda_T = c\hbar/k_B T$. Assume that the free charge carriers in one graphene sheet move with drift velocity $v \ll c$ (c is the light velocity) relative to the other graphene sheet. According to Ref.[31] the Casimir force F_z between moving media is determined by

$$F_z = F_{zT} + F_{z0}, \tag{1}$$

where the temperature dependent term F_{zT} and the zero-temperature contribution F_{z0} are given by

$$F_{zT} = \frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q e^{-2qd} \Bigg\{ \int_0^\infty d\omega \left(\frac{\mathrm{Im}R_1(\omega)\mathrm{Re}R_2(\omega^+)n_1(\omega) + \mathrm{Re}R_1(\omega)\mathrm{Im}R_2(\omega^+)n_2(\omega^+)}{|\,1 - e^{-2qd}R_1(\omega)R_2(\omega^+)\,|^2} \right.$$

$$+ (1 \leftrightarrow 2) \Bigg) + \int_0^{q_x v} d\omega \left(\frac{\mathrm{Re}R_1(\omega^-)\mathrm{Im}R_2(\omega)n_2(\omega)}{|\,1 - e^{-2qd}R_1(\omega^-)R_2(\omega)\,|^2} + (1 \leftrightarrow 2) \right) \Bigg\}, \tag{2}$$

$$F_{z0} = \frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x \Bigg\{ \mathrm{Re} \int_0^\infty d\omega s e^{-2sd} \left(\frac{R_1(i\omega)R_2(i\omega + q_x v)}{1 - e^{-2sd}R_1(i\omega)R_2(i\omega + q_x v)} \right.$$

$$+ (1 \leftrightarrow 2) \Bigg) + \int_0^{q_x v} d\omega q e^{-2qd} \left(\frac{\mathrm{Im}R_1(\omega)\mathrm{Re}R_2(\omega^-)}{|\,1 - e^{-2qd}R_1(i\omega)R_2(\omega^-)\,|^2} + (1 \leftrightarrow 2) \right) \Bigg\}, \tag{3}$$

where $n_i(\omega) = [\exp(\hbar\omega/k_B T_i - 1]^{-1}$ ($i = 1,2$), $q = \sqrt{q_x^2 + q_y^2}$, $s = \sqrt{(\omega/c)^2 + q^2}$, T_i is the temperature of i-th graphene sheet, R_i is the reflection amplitude for surface i for p-polarized electromagnetic waves, and $\omega^\pm = \omega \pm q_x v$. The symbol $(1 \leftrightarrow 2)$ denotes the terms that are obtained from the preceding terms by permutation of 1 and 2. In the first term in Eq. (3) the integration along the real axis was transformed into integration along the imaginary axis.

The reflection amplitude for a 2D-system is determined by[33]

$$R_i = \frac{\epsilon_i - 1}{\epsilon_i + 1}, \quad \epsilon_i = \frac{4\pi p \sigma_i}{\omega \varepsilon} + 1, \tag{4}$$

where $p = \sqrt{(\omega/c)^2 - q^2}$, σ_i is the longitudinal conductivity of the sheet which can written in the form $\sigma_i = -i\omega e^2 \Pi_i(\omega, q)/q^2$ where Π_i is the 2D polarizability, ε is the dielectric constant for the surrounded medium. The dielectric function of the sheet is determined by $\varepsilon_i(\omega, q) = 1 + v_q \Pi_i(\omega, q)$, $v_q = 2\pi e^2/q\varepsilon$ is the 2D Coulomb interaction. In term of ε_i the reflection amplitude can be written as

$$R_i = \frac{p(\varepsilon_i - 1)}{p(\varepsilon_i - 1) + iq} \tag{5}$$

In the integration on the real axis $p \approx iq$ for $d < \lambda_T$. Thus, in this case

$$R_i \approx \frac{\varepsilon_i - 1}{\varepsilon_i}, \tag{6}$$

On the imaginary axis $p = is$. In the finite lifetime generalization according to the Mermin approximation[34] the dielectric function is determined by

$$\varepsilon(\omega, q) \approx 1 + \frac{(\omega + i\gamma)(\varepsilon_0(\omega + i\gamma, q) - 1)}{\omega + i\gamma(\varepsilon_0(\omega + i\gamma, q) - 1)/(\varepsilon_0(0, q) - 1)}, \tag{7}$$

where $\varepsilon_0(\omega, q)$ is the RPA dielectric function.

In the study below we used the dielectric function of graphene, which was calculated recently within the random-phase approximation (RPA)[35,36]. The small (and constant) value of the graphene Wigner-Seitz radius r_s indicates that it is a weakly interacting system for all carries densities, making the RPA an excellent approximation for graphene (RPA is asymptotically exact in the $r_s \ll 1$ limit). The dielectric function is an analytical function in the upper half-space of the complex ω-plane:

$$\varepsilon_0(\omega, q) = 1 + \frac{4k_F e^2}{\hbar v_F q} - \frac{e^2 q}{2\hbar \sqrt{\omega^2 - v_F^2 q^2}} \left\{ G\left(\frac{\omega + 2v_F k_F}{v_F q} \right) - G\left(\frac{\omega - 2v_F k_F}{v_F q} \right) - i\pi \right\}, \tag{8}$$

where

$$G(x) = x\sqrt{x^2 - 1} - \ln(x + \sqrt{x^2 - 1}), \tag{9}$$

where the Fermi wave vector $k_F = (\pi n)^{1/2}$, n is the concentration of charge carriers, the Fermi energy $\epsilon_F = \gamma k_F = \hbar v_F k_F$, $\gamma = \hbar v_F \approx 6.5$ eVÅ, and v_F is the Fermi velocity. The damping parameter γ is due to scattering against impurities and acoustic phonons in graphene sheet, and can be expressed through the low field mobility μ: $\gamma = e v_F/(\hbar k_F \mu)$. Scattering of the graphene carries by the acoustic phonons of graphene places an intrinsic limits on the low-field room temperature ($T_0 = 300$ K) mobility, given by $\mu_0 = 20$ m^2/Vs at the graphene carriers density 10^{16} m^{-2} (see Ref.[37]), which gives $\gamma = 8 \cdot 10^{11}$ s^{-1}. At other temperatures the mobility can be obtained using the relation $\mu = \mu_0 T_0/T$.

In addition to the intrinsic friction due to scattering against impurities and phonons, on the electrons moving in the graphene sheet acts the extrinsic friction due to the interaction with electrons in the nearby graphene sheet. According to the theory of the Casimir friction[31], the friction force $F_x = F_{xT} + F_{x0}$, where at $d \ll \lambda_T$ and $v \ll c$ the contributions from thermal (F_{xT}) and quantum (F_{x0}) fluctuations are given by[38–40]

$$F_{xT} = \frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2qd} \left\{ \int_0^\infty d\omega \left(\frac{\mathrm{Im}R_1(\omega)\mathrm{Im}R_2(\omega^+)}{|\, 1 - e^{-2qd}R_1(\omega)R_2(\omega^+)\,|^2} \times [n_d(\omega) - n_g(\omega^+)] + (1 \leftrightarrow 2) \right) \right.$$

$$\left. - \int_0^{q_x v} d\omega \left(\frac{\mathrm{Im}R_d(\omega)\mathrm{Im}R_g(\omega^-)}{|\, 1 - e^{-2qd}R_d(\omega)R_g(\omega^-)\,|^2} n_1(\omega) + (1 \leftrightarrow 2) \right) \right\}, \tag{10}$$

$$F_{x0} = -\frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2qd} \int_0^{q_x v} d\omega \left(\frac{\mathrm{Im}R_1(\omega)\mathrm{Im}R_2(\omega^-)}{|\, 1 - e^{-2qd}R_1(\omega)R_2(\omega^-)\,|^2} n_1(\omega) + (1 \leftrightarrow 2) \right). \tag{11}$$

Due to the presence of an exponential factor in the expression (2) for the thermal contribution to the Casimir force, the integration over frequency is effectively limited to $\omega < \omega_T = k_B T/\hbar$. Thus for $q_x v \sim v/q > \omega_T$ (at room temperature and for $d = 1$ nm this condition corresponds to the velocities $v > 10^5$ m/s) the integrand will be modified in the whole range of integration, which will give rise to the significant change of the thermal Casimir force. This change will be especially large in the case of resonant photon tunneling when the integrand has sharp resonances. In the case of graphene the frequency of the emitted photons $\omega_{ph} > q_x v$. Resonance occurs when $\omega_{ph} > q_x v$ becomes equal to the creation energy of electron-hole pair in graphene $\omega_{eh} = v_F q$, which corresponds to $v > \sqrt{2}v_F \approx 1.4 \cdot 10^6$ m/s. The integrand in the expression for the zero-temperature contribution to the Casimir force does not contain any sharp cut-off in the frequency integration. Thus the range of integration will be more wide and the change of the zero-temperature contribution will be significant only for much higher velocities then for the thermal contribution.

Fig. 1a shows the dependence of the Casimir force between two graphene sheets on the separation d between the sheets. The thermal and quantum contributions are shown separately. The thermal contribution was calculated for $T = 600$ K and for the drift velocities $v = 0$ and $v = 2 \cdot 10^6$ m/s. The thermal contribution becomes larger then the quantum contribution for $d > 50$ nm. For $d < 5$ nm the thermal contribution calculated for $v = 2 \cdot 10^6$ m/s is significantly larger then the thermal contribution calculated at $v = 0$. For example, at $d \approx 3$ nm the drift motion of
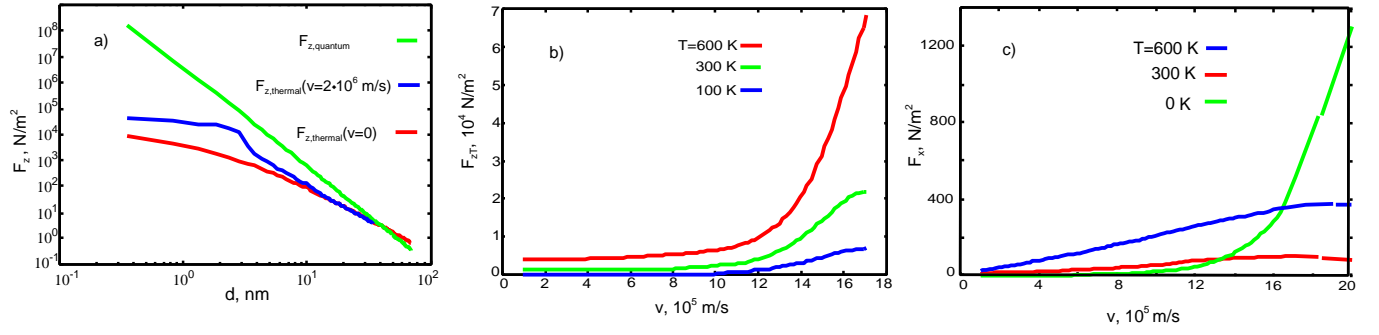
FIG. 1: The Casimir forces between two graphene sheets with carrier concentration $n = 10^{16} \mathrm{m}^{-2}$. (a) The dependence of the Casimir force on the separation d between the sheets. The thermal and quantum contributions to the total Casimir force are shown separately. The thermal contribution is shown for $T = 600$ K and for the drift velocities $v = 0$ and $v = 2 \cdot 10^6$ m/s. (b) The dependence of the thermal Casimir force and (c) the Casimir friction force on the drift velocity of electrons in graphene sheet at $d = 1$ nm. The non-zero temperature curves show only the thermal contribution to the friction.

the electrons gives rise to the increase of the thermal Casimir force by one order of magnitude, and in this case the thermal contribution is only one order of magnitude smaller then the quantum contribution. Figures 1b and 1c show the dependence of the thermal Casimir force F_{zT} and the Casimir friction force on the drift velocity of electrons in graphene sheet at $d = 1$ nm. Both of these forces are strongly enhanced in the case of resonant photon tunneling for $v > 1.4 v_F \approx 1.4 \cdot 10^6$ m/s, in accordance with the above theoretical arguments. For such case the Casimir friction force is dominated by quantum friction, existence of which was recently hot debated[41–46]. For two graphene sheets the Casimir friction force can be measured in frictional drag experiment. Such experiment was proposed theoretically some years ago[47,48] and performed experimentally for 2D-quantum wells[49,50]. As discussed in Ref.[40] quantum friction can be also detected by measuring the transport properties of non-suspended graphene on a SiO$_2$ substrate.

Concluding remarks.–We have calculated the dependence of the thermal Casimir force between two graphene sheets on the drift velocity of electrons in one graphene sheet. We have found that the drift motion of the electrons in graphene produces effects in the thermal Casimir force which can be measured experimentally. The thermal Casimir force as well as the Casimir friction force are strongly enhanced in the case of resonant phonon tunneling. For resonant photon tunneling for temperatures above room temperature the Casimir friction is dominated by quantum friction due to quantum fluctuations. Quantum friction can be detected in friction drag experiment between graphene sheets. Another way to detect quantum friction consists in measuring of the transport properties of nonsuspended graphene on an SiO$_2$ substrate in the high electric field.

[1] H.B.G.Casimir, Proc. K. Ned. Akad. Wet., **51**, 793 (1948).

[2] E.M.Lifshitz, Zh. Eksp. Teor. Fiz., **29**, (1955) [Sov. Phys. JETP **2**,73 (1956)]

[3] S. M. Rytov, *Theory of Electrical Fluctuation and Thermal Radiation* (Academy of Science of USSR Publishing, Moscow, 1953)

[4] M. L. Levin and S. M. Rytov, *Theory of eqilibrium thermal fluctuations in electrodynamics* (Science Publishing, Moscow, 1967)

[5] S. M. Rytov, Y. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophyics*(Springer, New York.1989), Vol.3

[6] I.E.Dzyaloshinskii, E.M.Lifshitz and L.P.Pitaevskii, Adv.Phys. **10**, 165 (1961)

[7] S.K.Lamoreaux, *Phys.Today* **60**, 40 (Februrary,2007)

[8] P.W.Milloni, *The Quantum Vacuum: An Introduction to Quantum Electrodynamics* (Academic,1993)

[9] A.O.Sushkov, W.J.Kim, D.A.R.Dalvit and S.K.Lamoreaux, Nature Phys. **7**, 230 (2011).

[10] F.M.Serry, D.Walliser and G.J.Maclay, J. Appl. Phys. **84**, 2501 (1998).

[11] E.Buks and M.L.Roukes, Phys.Rev.B **63**, 033402 (2001).

[12] G.L.Klimchitskaya, U.Mohideen and V.M.Mostepanenko, Rev. Mod. Phys. **81**, 1827 (2007).

[13] J.N.Munday, F.Capasso and V.A.Parsegian, Nature (London) **457**, 170 (2007).

[14] A.O.Sushkov, W.J.Kim, D.A.R.Dalvit and S.K.Lamoreaux, Phys. Rev. Lett. **107**, 171101 (2011).

[15] Y.Bao, R.Guérout, J.Lussange, A.Lambrecht, R.A. Cirelli, F.Klemens, W.M.Mansfield, C.S.Pai and H.B.Chan, Phys. Rev. Lett. **105**, 250402 (2010).

[16] A.W.Rodriguez, W.J.Kim, F.Capasso and S.G.Johnson, Nature Photon. **5**, 211 (2011).

[17] R.Zhao, J.Zhou, Th.Koschny, E.N.Economou and C.M.Soukoulis, Phys. Rev. Lett. **103**, 103602 (2009).

[18] M.Bordag, B.Geyer, G.L.Klimchitskaya and V.M.Mostepanenko, Phys.Rev.B **74**, 205431 (2006).

[19] M.Bordag, I.V.Fialkovsky, D.M.Gitman and D.V.Vassilevich, Phys.Rev.B **80**, 245406 (2009).

[20] D.Drosdoff and L.M.Woods, Phys.Rev.A **84**, 062501 (2011).

[21] D.Drosdoff and L.M.Woods, Phys.Rev.B **82**, 155459 (2010).

[22] I.V.Fialkovsky, V.N.Marachevsky and D.V.Vassilevich, Phys.Rev.B **84**, 035446 (2011).

[23] B.E.Sernelius, *EPL* **95**, 57003 (2011).

[24] V.Svetovoy, Z.Moktadir, M.Elwenspoek and H.Mizuta, EPL **96**, 14006 (2011).

[25] J.Sarabadani, A.Naji, R.Asgari and R.Podgornik, Phys.Rev.B **84**, 155407 (2011).

[26] G.Gómez-Santos, Phys.Rev.B **80**, 245424 (2009).

[27] K.S. Novoselov, A.K. Geim, S.V. Morosov, D. Jiang, Y. Zhang, S.V. Dubonos, I.V. Grigorieva, and A.A. Firsov, Science **306**, 666 (2004).

[28] K.S. Novoselov, S.V. Morosov, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, and A.A. Firsov, Nature (London) **197**, 197 (2005).

[29] A.K. Geim and K.S. Novoselov, Nat. Mater. **6**, 183 (2007).

[30] J.B. Pendry, J. Phys.C, **9**, 10301 (1997).

[31] A.I.Volokitin and B.N.J. Persson, Phys. Rev. B **78**, 155437 (2008); *ibid.* **81**, 239901(E) (2010).

[32] A.I.Volokitin and B.N.J. Persson, New J. Phys. **13**, 068001 (2011).

[33] A.I.Volokitin and B.N.J. Persson, J.Phys.: Condens. Matter **13**, 859 (2001).

[34] N.D.Mermin, Phys. Rev. B **1**, 2362 (1970).

[35] B. Wunscvh, T. Stauber, F. Sols, and F. Guinea, New J.Phys. **8**,318 (2006).

[36] E.H. Hwang, S.Das Sarma, Phys. Rev. B **75**, 205418 (2007).

[37] J.H. Chen, C. Jang, S. Xiao, M. Ishigami and M.S.Fuhrer, Nat. Nanotechnol. **3**, 206 (2008)

[38] A.I.Volokitin and B.N.J. Persson, J.Phys.: Condens. Matter **11**, 345 (1999); Phys.Low-Dim.Struct. **7/8**, 17 (1998).

[39] A.I.Volokitin and B.N.J. Persson, Rev. Mod. Phys. **79**, 1291 (2007).

[40] A.I. Volokitin and B.N.J. Persson, Phys. Rev. Lett.**106**, 094502 (2011).

[41] T.G. Philbin and U. Leonhardt, New J. Phys. **11**,033035 (2009).

[42] J.B. Pendry, New J. Phys. **12**, 033028 (2010).

[43] U. Leonhardt, New J. Phys. **12**,068001 (2010).

[44] J.B. Pendry, New J. Phys. **12**, 068002 (2010).

[45] A.I.Volokitin and B.N.J. Persson, New J. Phys. **13**, 068001 (2011).

[46] T.G.Philbin, and U.Leonhardt, New J. Phys. **13**, 068002 (2011).

[47] M.B. Pogrebinskii, Fiz.Tekh.Poluprov. **11**, 637 (1977) [Sov.Phys. Semicond. **11**, 372 (1977)].

[48] P. J. Price, Physica B+C **117**,750 (1983).

[49] T.J. Gramila, J.P. Eisenstein, A.H. MacDonald, L.N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **66**, 1216 (1991).

[50] U. Sivan, P.M. Solomon, and H. Shtrikman, Phys. Rev. Lett. **68,** 1196 (1992).